Mathematical modeling and linear systems – Student Handout

Our Goal: To interactively understand a real-world situation and then connect this to the practical use of solving linear systems in mathematics.

Problem situation: A boat travels downstream and upstream on a river with a current. We assume that – if the boat was in still water – its speed would be constant. We also assume that the speed of the current of the river is constant. **How does the speed of the current affect that actual speed of the boat as it travels downstream and as it travels upstream?**

Open the interactivity: BoatCurrent (online) **or boatcurrent.html** (offline).

Getting started: We are going to test the boat using the simulator. But first let's consider.
Let v_b = the speed of the boat in still water. Let v_r = speed of the river current (positive ☺).
Let v_D = speed of the boat downstream. Let v_U = speed of the boat upstream.

- Fill in the squares at right with v_b, v_D and v_U. ☐ < ☐ < ☐

Understanding the simulator – interactivity

- Click on the Start/Stop button. As time advances (top left), the boat and the driftwood will move in the river with the current and the boat will move in still water.
- Click on the Start/Stop button again. The simulator will stop. Click on the Reset button and the boats and driftwood will return to their starting zero position.
- Click and drag the **purple point on the purple slider**. This changes the direction of the boats.
- Click and drag the **red point on the red slider** to change the speed of the boat in still water and on the yellow point on the yellow slider to change the speed of the river current.
- Freely change these speeds, the directions of the boats and test the simulator several times.

Downstream

- **Click on the  button at the top-right corner of the simulator to reset all values.**
- Click again on the start/stop button and now let the animation finish. Notice **Time =5 hours**.
- Find the distance traveled by the boat in still water in these 5 hours. Calculate its speed v_b. Does your answer match the slider speed at the top (just right of time)?
- Find the distance traveled by the driftwood. Calculate its speed v_r. Does it match the yellow slider? What can you say about the speed of the driftwood and the speed of the current?
- Finally, find the distance traveled by the boat downstream. Relate this distance, the distance traveled by the boat in still water and the distance traveled by the driftwood.
- Calculate the speed v_D of the boat going downstream. What is the **relationship** between this speed, the speed of the boat in still water and the speed the river current. Write this relationship as an equation using v_b, v_r and v_D and **circle your equation** so you can find it.
- Click and drag sliders to change the speed of the boat in still water v_b = **50 km/h** and of the river current v_r = **15 km/h**. Using the formula: $s_D = (v_b + v_r) \cdot t$ calculate the distance the boat will travel downstream in **3 hours**.
- Click on Manual Animation button. You will see a slider underneath the time. Click and drag this slider to time **t=3 hours**. Find the distance traveled by the boat going downstream. Does this agree with your calculation?
- Calculate this boats speed v_D. Does your equation using v_b, v_r and v_D work here? It should.

Upstream

- If you want, change the slider values for speed of boat in still water v_b and of current v_r.
- Click and drag the slider at the right so that the boat in the river is going upstream. (Remember that even though the boat in still water will go in the opposite direction as before, this doesn't affect its speed since direction doesn't matter in still water.)
- Click on the reset button or click and drag the manual animation slider to **t = 0**.
- Run the animation – click on the Start/Stop button or use the manual animator.
- Find the distance traveled by the boat going upstream. Calculate its speed v_U. What is the **relationship** between this speed, the speed of the boat in still water and the speed of the current. Write an equation using v_b, v_r and v_U and **circle your equation** so you can find it

Modeling – translating real-world to mathematics, solving and translating back

Problem: A boat travels downstream on a river at 60 km/h. It travels upstream on the same river at 40 km/h. What is the speed of the boat in still water and the speed of the current of the river? Assume that the speed of the boat in still water and speed of the current of the river are constant.

- What are the values of v_b, v_r, v_D and v_U? If they are unknown variables, write " = ? "
- Using your **circled equations** from above, make a system of 2 equations in 2 unknown variables. Solve this system and write down your answer.
- Rewrite this answer as an answer that fits the question – write in people language.
- Think a bit about how you would check your answer on the simulator. What should be the values on the sliders for v_b and v_r? Change the sliders to these values.
- Run the simulator **downstream** for t=1 hour (manually). If your answer is correct, how far should the boat going downstream travel? If you can, take a screenshot and explain how this shows that your answer is correct.
- Run the simulator **upstream** for t=1 hour (manually). If your answer is correct, how far should the boat going upstream travel? If you can, take a screenshot and explain it.

A little challenge

Problem: A boat went 20km downstream in 20 minutes. It then went upstream 40km in 2 hours. At what speed would a piece of driftwood move in this river? Assume that the speed of the boat in still water and speed of the current of the river are constant.

Solve this problem and check your answer on the simulator. Take screenshots of your 2 checks (downstream and upstream) and explain them.

You make a question!

Set the speed of the boat in still water and the speed of the river current to numbers you like.

1. Set the purple slider for downstream and manually move the animator to a time of your choosing. Check the distance the boat went downstream. Write a sentence about this.
2. Set the slider for upstream and again move the animator to a time of your choosing. (Don't change the speeds!) Check the distance the boat went upstream. Write a sentence about this.
3. Give these 2 sentences to your friends. See if they can solve the question: At what speed would a piece of driftwood move in this river?