Triangles - Medians and RightTriangles

Theorem: Let $\overline{CM}$ be the ***median*** to the longest side $\overline{AB}$ of a triangle ABC.
That is, $\overline{CM}$ is the line segment joining the point C to the *midpoint* of $\overline{AB}$.
If $\overline{CM}$ is exactly **half** the length $\overline{AB}$ *then* ΔABC is a **right-triangle**.

Open the interactivity: tri_median_rightA.ggb

- Click and drag the slider a to change the length of the side
- Click and drag the slider m to change the length of the median (and the hypotenuse).
- Click and drag point C to move and point A to rotate the triangle.
- Select the checkbox to show triangle ΔABC.

Notice that angle $\angle BCA$ appears to be 90° (a right-angle).

Questions to think about?

- Does triangle ΔABC always satisfy the theorem (no matter the slider or point positions)?
- **Select checkbox: Show Angles**

1. How many degrees in: $\alpha+\beta$?
2. What kind of triangle is ΔAMC ?
3. How many degrees is: $2\delta+\beta$?
4. Why does $\frac{\delta}{2}=\alpha$?
5. What kind of triangle is ΔBMC ?
6. Why does $\frac{\alpha}{2}+\gamma=90°$?

- Now, why does angle $\angle BCA = 90°$.

Open Interactivity: tri_median_rightB.ggb and make the construction!

- First study the construction in the above interactivity. Then, make your construction.

When you are done, check that you can move the sliders and points A and C and that triangle ΔABC still "works" (satisfies the theorem)!

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Brief	User interacts with construction and notes that the triangle "appears" to be a right triangle. Then, using only the fact that the lower angles of an isosceles triangle are equal, the user "proves" that the angle is in fact 90°. Finally, the student constructs the same interactivity.
Goal	Understanding triangles.
Grade	6-9 (6th grade, 7th grade, pre-algebra, geometry)
Strand	Measurement and Geometry, Geometry
Standards	CA 6.MG.2.2, CA Geometry 13.0, ACT PF 24-27
Keywords	triangles, medians, isosceles, right-triangles, construction, interactivity, geogebra
Comments	Suitable for 6th-grade on up.
Source	Linda Fahlberg-Stojanovska (no copyright)
Cost	Activity and software is free to use
Download	Requires freeware GeoGebra (www.geogebra.org) for offline use.
Type	Java Applet so requires free sunJava player
Files	tri_median_right.pdf (or doc), tri_median_rightA.ggb, tri_median_rightB.ggb

The construction steps

1. Click on  , then on point C and then type in "a" (letter a - no quotes) and hit Enter.
2. Click on  , then on any point on the circle to get point A.
3. Click on  , then on point C and then on point A.

? Question? What one icon could we use to do steps 1-3 all at one time ? Answer: 

4. Click on  , then on point C and type in "m" …
5. Click on  , then on point A and type in "m" …
6. Click on  , then on the intersection of circles from 4 and 5.
   - Rename this point M (otherwise third point of main triangle will be D).
7. Click on  , then on point A and then on point M
8. Click on  , then on point M and type in "m" …
9. Click on  , then on the intersection of ray from 7 and circle from 8 to get point B.
10. Click on  , then on point B and then on point C.
    - Hide circles and ray.
11. Click on  , then on A, then C and then B to check that it is 90°.

Other questions

Do you think that if ΔABC is a right triangle that M will be half of $\overline{AB}$?

Answer: It is, but this is a much harder proof. (See e.g. here.)