Vertical Motion – Free Fall (worksheet under construction)

Theorem: The height of an object at time *t* in free fall is: $h(t) = h_o + v_o t - 16t^2$ [ft].
The initial height is h_o [ft] and the initial velocity is v_o [ft/s].

Open the interactivity: motion_v3.html

- Change the unit to feet (default) or meters at the bottom left of screen.
- Click on the start/stop button to start and stop the animation. Notice that the object (in this case a cannon ball) moves along the y-axis in a vertical up and down motion. The horizontal axis is ***time*** – nothing is moving horizontally!
- Test the other animation buttons. Click on the reset button to restart the animation. Click on the slower and faster buttons to slow down or speed up the animation. Finally click on the step backward and step forward buttons to advance time forward and backward.

Problems first or later

- Set $h_o = 80$ [ft] and $v_o = -20$ [ft/s]. Run the animation. Point with your finger to the place **where** the ball hit the ground. Now find the place on the graph where it says **when** it hit the ground.
- Set $h_o = 80$ [ft] and $v_o = 20$ [ft/s]. Run the animation. Notice that the ball goes up before it goes down. Why is this? Reset the animation and using the step forward and step backward buttons stop the animation when the ball is at its highest point. Point with your finger to the place where the ball is at its highest point. Now find the place on the graph where it says when it is at its peak. What time is this?
- Given $h_o = 100$ [ft] and $v_o = 0$ [ft/s] , find when the ball hits the ground. Do this via the animation and algebraically using the function. When is the ball at its highest point? What is this highest point? Do not forget units!
- Given $h_o = 0$ [ft] and $v_o = 100$ [ft/s], find when the ball hits the ground. Do this via the animation and algebraically using the function. When is the ball at its highest point (remember – parabolas are symmetric!)? What is this highest point? Do not forget units!
- Given $h_o = 100$ [ft] and $v_o = 100$ [ft/s], find when the ball hits the ground. Do this via the animation and algebraically using the function. When is the ball at its highest point? What is this highest point? Do not forget units!

  If you are standing at h_o, when will the ball pass you?

Function questions to think about?

- The function $h(t)$ gives height in [ft]. So each member of the function must give [ft]. Remember that h_o is (initial) height. So its unit is [ft]. It is all by itself so this is good.

- v_o is (initial) velocity. So its unit is [ft/s]. How does this member give [ft]?
- What do you think the unit of "16" is so that this last member gives [ft]?

- In what part of the function is gravity playing a part? In which of the above problems is the only force gravity?
- Why do you think there is a plus sign in front of v_o and a minus sign in front of 16? That is, what does it mean in mathematics/physics for an object to have a positive velocity? Does gravity increase this velocity?
- When you start the simulation $h_o = 80\,[\text{ft}]$ and $v_o = -20\,[\text{ft/s}]$. Make up a problem that describes this situation.
- The word "initial" means at time $t = 0$. Substitute $t = 0$ into the function. What do you get? Does this agree with the statement at the top that h_o is the initial height?
- Why do you think we use the words "initial velocity" to describe v_o? Does the velocity of the object change?

Animation questions to think about?

- What is the velocity of the ball at its highest point?
- Reload/refresh the page from the browser! Set the initial height to 0 and the initial velocity to 100. Click **exactly** 3 times on Faster. Now run the animation completely. The points at the top of the parabola should be closer together than the points at the bottom of the parabola. Why do you think this happens? In what sense are the points "closer" together?

You make a Simulator:

- Make sliders h0 and v0 for h_o and v_o. By default they will take values between -5 and 5 at increments of 0.1. Adjust them "realistically".
- Make a slider for *t*.
- Define a function of f(x) that is the height of the object at time x.
- Define points H=(0,f(t)) and F=(t,f(t)). What is H and what is F?
- Move the slider t and adjust its minimum, maximum and increment so that the results have the precision your teacher requires.
- You can put an animation on t – remember to change it to "increasing".
- You can put a trace on F but remember this is NOT the path of the ball!

Calculus questions to think about?

- At the beginning we stated the height *h* as a function of *t*. The average speed or velocity of the ball during an interval of *t* would be the "change in *h*"/"change in *t*".

  Use calculus to find a formula/function describing the exact velocity of the object at time *t*.

- So – using calculus – we see that $v(t) = v_o - 32t$ [ft/s].
  - When scientists started thinking about this, which function do you think "came first" – $v(t)$ or $h(t)$? Why might you think this?
  - Using words, state exactly what "32" is using the correct units.
  - Suppose that you threw a ball straight up and then caught it at the same height 3.2s later. What is the speed of your throw? Can you determine how high it went up from where you threw it?
  - Now suppose you are on the planet Dusto and you threw a ball straight up with the same speed and it came down 4.7s later. What facts can you determine from this? What do you think - is Dusto bigger or smaller than Earth?