Vertical Motion – Free Fall

Theorem: The height of an object at time t in free fall is: $h(t) = h_o + v_o t - 16t^2$ [ft].
The initial height is h_o [ft] and the initial velocity is v_o [ft/sec].

Open the interactivity (online or download: school or offline):

Change the unit to feet (default) or meters.

Click on the start/stop button to start and stop the animation. Notice that the object (in this case a cannon ball) moves along the y-axis in a vertical up and down motion. The horizontal axis is ***time*** – **nothing is moving horizontally**!

Test the other animation buttons. Click on the reset button to restart the animation. Click on the slower and faster buttons to slow down or speed up the animation. Finally click on the step backward and step forward buttons to advance time forward and backward step-by-step.

Problems first or later

Q1. Set $h_o = 80$ [ft] and $v_o = -20$ [ft/s]. Run the animation until the ball hits the ground. Point with your finger to the place **where** the ball hits the ground. Now find the place on the graph where it says **when** it hit the ground.

Q2. Set $h_o = 80$ [ft] and $v_o = 20$ [ft/s]. Run the animation. Notice that now the ball goes up before it goes down. Why is this? Reset the animation and using the step forward and step backward buttons stop the animation when the ball is at its highest point. Point with your finger to the place where the ball is at its highest point. Now find the place on the graph where it says **when** it is at its peak. What time is this?

Q3. Given $h_o = 100$ [ft] and $v_o = 0$ [ft/s] , find when the ball hits the ground. Do this via the animation and algebraically using the function $h(t)$. When is the ball at its highest point? What is this highest point? Make sure every answer includes units!

Q4. Given $h_o = 0$ [ft] and $v_o = 100$ [ft/s], find when the ball hits the ground. Do this via the animation and algebraically using the function $h(t)$. When is the ball at its highest point (remember – parabolas are symmetric!)? What is this highest point? Do not forget units!

Q5. Given $h_o = 100$ [ft] and $v_o = 100$ [ft/s], find when the ball hits the ground. Do this via the animation and algebraically using the function $h(t)$. When is the ball at its highest point? What is this highest point? Do not forget units!

If you are standing at height h_o, when will the ball pass you?

Very easy solving but some thinking about why …

Q6. Little Julie throws a ball vertically up from a height of 2 ft off ground with a speed of 16 ft/s. Andrew is standing next to Julie and he catches the ball when it is on its way down at

the same height of 2 ft from the ground. Use your knowledge of math and the function h(t) to find out how much time passed between when Julie throws and Andrew catches the ball. Now test your answer with simulation by moving the time slider to the approximate time. Note that the way GeoGebra is set up it may not possible to get the very exact time that your theoretical answer gave but it will be close.

Q7. Now Andrew throws a ball vertically up from a height of 2ft off the ground and 5sec later catches it at the same height. In the simulator set the initial height to 2ft. Now think about how you manipulate the simulator to find out how fast Andrew throws the ball? Try to derive this same answer using the function $h(t)$.

Q8. For fun: Look back at the simulator. How high did the ball go? Do you think it is possible for a person to throw a ball that high?

Q9. Think about questions Q6 and Q7. Then fill in the following table.

Initial height	Initial speed	Time to return to initial height
2 ft	16 ft/s	
2 ft		5 sec
0 ft	16 ft/s	
0 ft	32 ft/s	
0 ft		3 sec
0 ft	64 ft/s	
0 ft		5 sec

Q10. Try to express this relationship in words and also as a function.

Function questions to think about?

Q11. The function $h(t)$ gives height in feet. This means that each of the three components h_0, $v_0 \cdot t$ and $-16 \cdot t^2$ must also express feet.

== The component h_0 is all by itself and h_0 is (initial) height so it's unit is feet.

== What is the unit of "v_0" so that $v_0 \cdot t$ expresses feet?

Is this a unit for speed? Name some other units for speed. Are they the same type d/t?

== What is the unit of "16" so that $-16 \cdot t^2$ expresses feet?

What kind of unit is this?

Q12. In what part of the function is gravity playing a part? Do you know what is the gravitational constant g on earth? Can you find it on the internet and write it down in both ft/s^2 and in m/s^2? (The *gravitational constant* is also called the *acceleration constant*.)

Q13. Experiments in Tucson ,Arizona show that a ball dropped (initial velocity 0 ft/sec) from a tower, after one second, will be traveling about 32 ft/s (10 m/s) after two seconds, it is traveling at about 64 ft/s, (20 m/s) and after three seconds, it is traveling at about 96 ft/s

(30 m/s). Do you think that this experiment depend on the location? Will the results be the same in other places? On the moon also? Explain. How is this connected to the components of the function h(t)? What will the function look like on Mars (check on the internet for the acceleration constant on Mars)?

Q14. Why do you think there is a plus sign in front of v_o and a minus sign in front of 16? That is, what does it mean in mathematics/physics for an object to have a positive velocity? Does gravity increase or decrease this velocity?

Q15. Imagine that you start the simulation with $h_o = 80\,[\text{ft}]$ and $v_o = -20\,[\text{ft/s}]$. Make up a problem that describes this situation.

Q16. The word "initial" means at time $t = 0$. Substitute $t = 0$ into the function. What do you get? Does this agree with the statement at the top that h_o is the initial height?

Q17. Why do you think we use the words "*initial* velocity" to describe v_o? Does the velocity of the object change?

Animation questions to think about?

Q18. What is the velocity of the ball at its highest point?

Q19. Reload/refresh the page from the browser! Set the initial height to 0 and the initial velocity to 100. Click **exactly** 3 times on Faster. Now run the animation completely. The points at the top of the parabola should be closer together than the points at the bottom of the parabola. Why do you think this happens? In what sense are the points "closer" together?

You make a Simulator:

Q20. Make sliders h0 and v0 for h_o and v_o. By default they will take values between -5 and 5 at increments of 0.1. Adjust them "realistically".

Q21. Make a slider for t.

Q22. Define a function of f(x) that is the height of the object at time x.

Q23. Define points H=(0,f(t)) and F=(t,f(t)). What is H and what is F?

Q24. Move the slider t and adjust its minimum, maximum and increment so that the results have the precision your teacher requires.

Q25. You can put an animation on t – remember to change it to "increasing".

Q26. You can put a trace on F but remember this is NOT the path of the ball!

Calculus questions to think about?

Q27. At the beginning we stated the height *h* as a function of *t*. The average speed or velocity of the ball during an interval of *t* would be the "change in *h*" / "change in *t*".

Use calculus to find a formula/function describing the exact velocity of the object at time *t*.

Q28. So – using calculus – we see that $v(t) = v_o - 32t$ [ft/s].

- When scientists started thinking about this, which function do you think "came first" – $v(t)$ or $h(t)$? Why might you think this?
- Using words, state exactly what "32" is using the correct units.
- Suppose that you threw a ball straight up and then caught it at the same height 1 second later. What was the speed of the ball as you threw it? (The answer is 16 ft/sec. which is half the gravitational constant g on Earth). What was the maximal height of the ball?
- Now suppose you are on the planet Dusto and you threw the ball up with the same speed, but the ball came down 2 seconds later. Do you weigh more on Dusto than you weigh on Earth? Explain. Did you use the same, less or more force to throw the ball up on Dusto than on Earth? Explain. Can you guess what the gravitational constant is on Dusto (note that on earth g =32 ft/sec2 and 32=16·2). Does Dusto have more mass than Earth or less?